Interpretable exemplar-based shape classification using constrained sparse linear models

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Motivation for shape analysis

Sagittal

Coronal

Axial

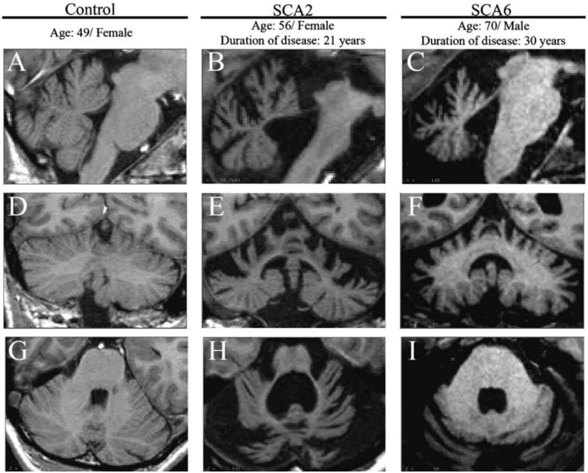

Figure : Atrophy in Spinocerebellar Ataxia[1]

[1] Brian C. Jung, e. a., "Principal component analysis of cerebellar shape on mri separates sca types 2 and 6 into two archetypal modes of degeneration," *Cerebellum* **11**, 887–895 (2012)

Motivation for interpretable shape analysis

- Shape changes in brain disorders:
  - Cerebellum (Spinocerebellar Ataxia)
  - Hippocampus-amygdala (Schizophrenia)

Can we create effective tools that allow clinicians to analyze shape as they would symptoms?

Can we make these tools intuitive to use?

Figure : 3D rendering of a cerebellum from MRI

What can we do to improve?

- *Problem:* Classifying shapes into categories (e.g. disease)
- Traditional Machine Learning approaches use features/keypoints
  - Gorelick *et al.*[1], Zhang *et al.*[2]

  How do we interpret a separator in a feature space?
- Most use parametric classifiers.
  - Golland *et al.*[3]

  Can we get more information by pointing to the training examples that led us to the conclusion?

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[1] Gorelick, L., Galun, M., Sharon, E., Basri, R., and Brandt, A., "Shape representation and classification using the poisson equation," *Pattern Analysis and Machine Intelligence, IEEE Transactions on* **28**(12), 1991–2005 (2006)

[2] Zhang, S., Zhan, Y., Dewan, M., Huang, J., Metaxas, D. N., and Zhou, X. S., "Towards robust and effective shape modeling: Sparse shape composition," *Medical image analysis* **16**(1), 265–277 (2012)

[3] Golland, P., Grimson, W. E. L., Shenton, M. E., and Kikinis, R., "Small sample size learning for shape analysis of anatomical structures," in [*Medical Image Computing and Computer-Assisted Intervention–MICCAI 2000*], 72–82, Springer (2000)

Method

We use non-parametric exemplar based classification similar to nearest neighbors

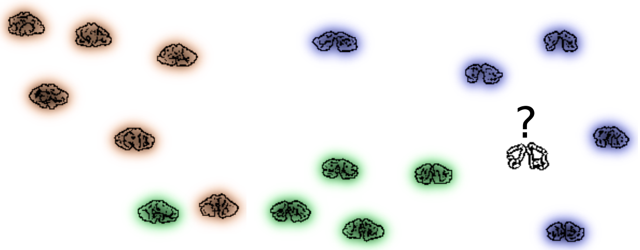


Figure : A space of shapes. The test shape belongs to the blue class.

Tells us which shapes we used to reach this conclusion, and how important they were.

First, at par with state of the art in 2D Classification

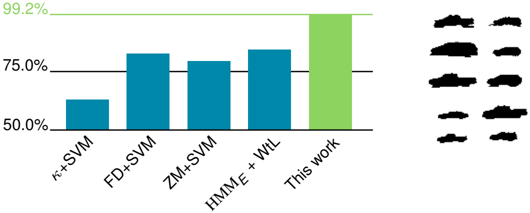


Figure : Classification accuracy for the vehicle dataset. [1]

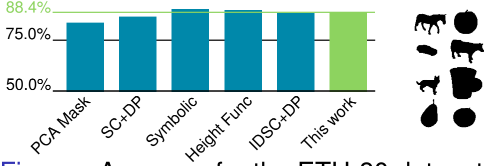


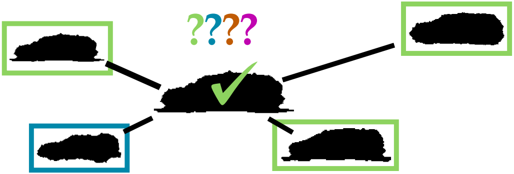
Figure : Accuracy for the ETH-80 dataset [2]

[1] Thakoor, N., Gao, J., and Jung, S., "Hidden markov model-based weighted likelihood discriminant for 2-d shape classification," *Image Processing, IEEE Transactions on* **16**(11), 2707–2719 (2007)

[2] Leibe, B. and Schiele, B., "Analyzing appearance and contour based methods for object categorization," in [*Computer Vision and Pattern Recognition, 2003. Proceedings. 2003 IEEE Computer Society Conference on*], **2**,

Intuition

- Finds shapes approximating the shape.
  - Uses those to find the class.



- In a clinical setting, an interface for analysis:

Next, how to we approximate?

Classification by sparse recovery

- Typical set up for compressed sensing/sparse recovery
- Dictionary of shapes $\Phi = [\phi_1, \phi_2, \ldots, \phi_K]$
- Find a collection of shapes that fits the test shape $\mathbf{y}$[1]

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_0 \ \text{ subject to } \ \Phi\mathbf{x} = \mathbf{y}$$

$$\hat{c} = \arg\min_{c \in \mathcal{C}} \|\mathbf{y} - \Phi\delta_c(\hat{\mathbf{x}})\|_2$$

- (shape classes $\mathcal{C}$, $\delta_c(\mathbf{x})$ zeros $\mathbf{x}$'s elements not in c)
- Related to work on sparse dictionaries for segmentation[2]

[1] Wright, J., Yang, A. Y., Ganesh, A., Sastry, S. S., and Ma, Y., "Robust face recognition via sparse representation," *Pattern Analysis and Machine Intelligence, IEEE Transactions on* **31**(2), 210–227 (2009)

[2] Zhang, S., Zhan, Y., Dewan, M., Huang, J., Metaxas, D. N., and Zhou, X. S., "Towards robust and effective shape modeling: Sparse shape composition," *Medical image analysis* **16**(1), 265–277 (2012)

How do we represent the shapes such that we can intuitively reason about the final result?

- Signed Distance Functions (SDF)
  - Distance to the shape boundary
  - Previously successful in shape classification[1]

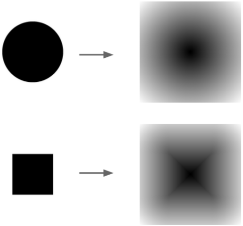
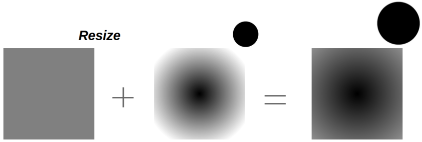


Figure : Example SDFs for two shapes

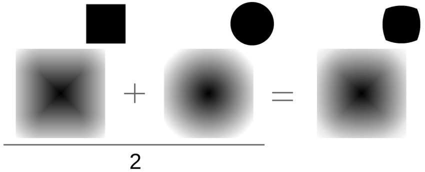
[1] Tsai, A., Wells, W. M., Warfield, S. K., and Willsky, A. S., "An em algorithm for shape classification based on level sets," *Medical Image Analysis* **9**(5), 491–502 (2005)

Operations on signed distance functions

- Resizing (Diluting/eroding with a circular element)
  - Adding/subtracting constant changes size
  - $\sum x_i\phi_i + k_i = \sum x_i\phi_i + \sum k_i$ (constants merge)



- Blending
  - If 50/50, new boundary is in middle of boundaries.



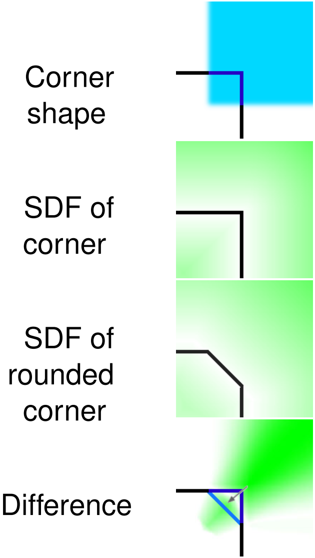
  - Previously used to define the "average shape"[1]

[1] Leventon, M. E., Grimson, W. E. L., and Faugeras, O., "Statistical shape influence in geodesic active contours," in [*Computer Vision and Pattern Recognition, 2000. Proceedings. IEEE Conference on*], 1, 316–323, IEEE (2000)

What properties emerge?

Lossless shape representation

- We minimize the difference between SDFs
  - Related to motions of the shape boundary
  - and size of boundary's influence zone (Blue)



Lossless shape representation

- What constraints on the optimization?
  - $\sum x_i = 1$
    - Assuming simple shapes: necessary for minimum
    - Necessary for outcome being SDF
  - $x_i \geq 0$
    - Avoids inside out shapes

That is, only convex combinations of shapes. (Regularizer)

$$\hat{\mathbf{x}} = \arg\min_{x} \|\Phi\mathbf{x} - \mathbf{y}\|_2 \text{ s.t. } \|\mathbf{x}\|_0 \leq s, \ \|\mathbf{x}\|_1 = 1 \text{ and } \mathbf{x} \geq 0$$

Using these properties, this tells us how we manipulated the shapes to come to our conclusion

Algorithm Summary

- Related to Orthogonal Matching Pursuit
- Convex constrained quadratic system solved with efficient quadratic programming
- Add constants to dictionary for invariance to scaling

Algorithm 1 Shape Classification using Sparse Convex Combinations

Require: $\mathbf{y}$ (Test shape), $\Phi = [\phi_1, \phi_2, \ldots, \phi_K]$ (Dictionary), s (Sparsity), $\mathcal{C}$ (Classes)

$\Phi = \Phi \cup \{\mathbf{1}, -\mathbf{1}\}$

$\mathbf{y}_r^0 = \mathbf{y}$, $S^0 = \emptyset$

for n=1 **to** s **do**

$i_{\max} = \arg\max_{i \subset [N] \setminus S} \langle \phi_i, \mathbf{y}_r^{n-1} \rangle / \|\phi_i\|_2$

$S^n = S^{n-1} \cup \{i_{\max}\}$

$\hat{\mathbf{x}} = \arg\min_x \|\Phi_{S^n}\mathbf{x} - \mathbf{y}\|_2$ s.t. $\|\mathbf{x}\|_1 = 1$ and $\mathbf{x} \geq 0$

$\mathbf{y}_r^n = \mathbf{y} - \Phi_{S^n}\hat{\mathbf{x}}$

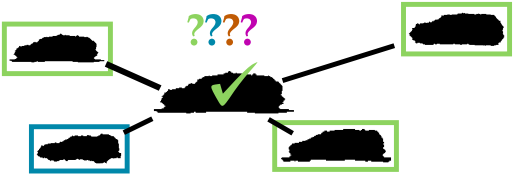
end for

return $\hat{c} = \arg\min_{c \in \mathcal{C}} \|\mathbf{y} - \Phi\delta_c(\hat{\mathbf{x}})\|_2$ (Class), Φ_{S^n} (Similar shapes), $\hat{\mathbf{x}}_{S^n}$ (Similarity weights)

(MATLAB implementation available online)

Intuition revisited

- Finds shapes approximating the shape.
  - Uses those to find the class.



- In a clinical setting, an interface for analysis:

Evaluation

3D Classification at par with state of the art

- 93 Subjects from 4 groups (Controls, and 3 types of Ataxia)

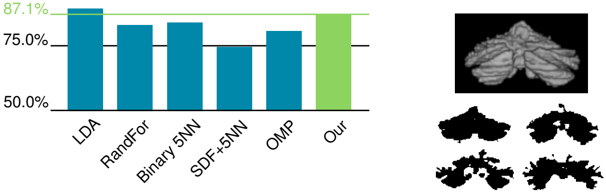


Figure : Classification accuracy for the cerebellum dataset.

Truth	Prediction: Controls	SCA2	SCA6	AT
Controls	100%	0%	0%	0%
SCA2	8.3%	83.3%	0%	8.3%
SCA6	14.3%	0%	71.4%	14.3%
AT	21.1%	0%	10.5%	68.4%

Table : Confusion matrix for the cerebellar disease classification task.

Summary

Effective computational shape analysis:

That everyone can use?

- Complete shape information
- Picks few shapes from dictionary to approximate shape
- Uses intuitive shape operations: Resizing and Blending
- At par with state-of-the-art

The End

- Thanks:
  - NIH 2R01NS056307
- Questions?