# Smoothing fields of weighted collections with applications to diffusion MRI processing


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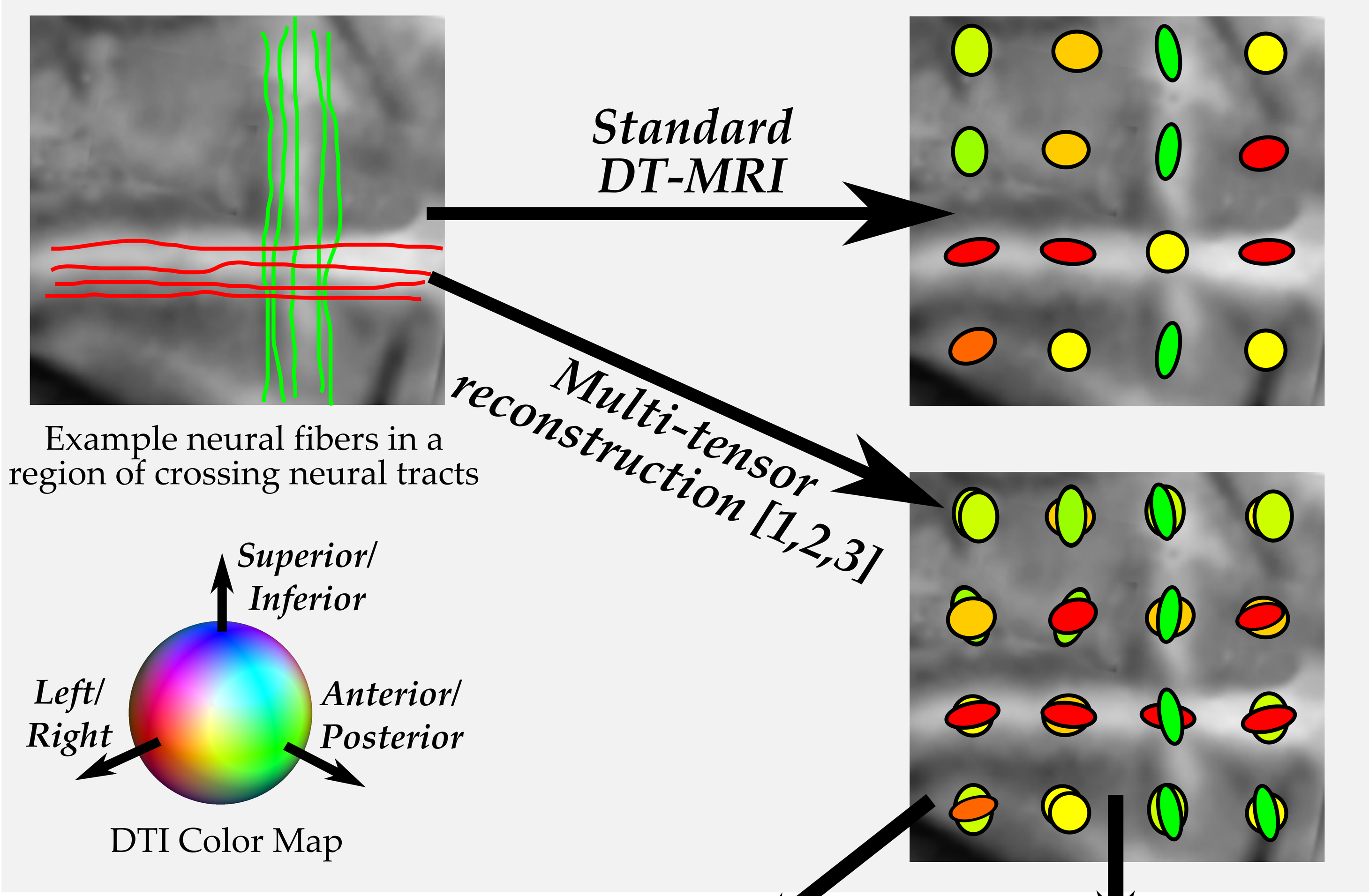
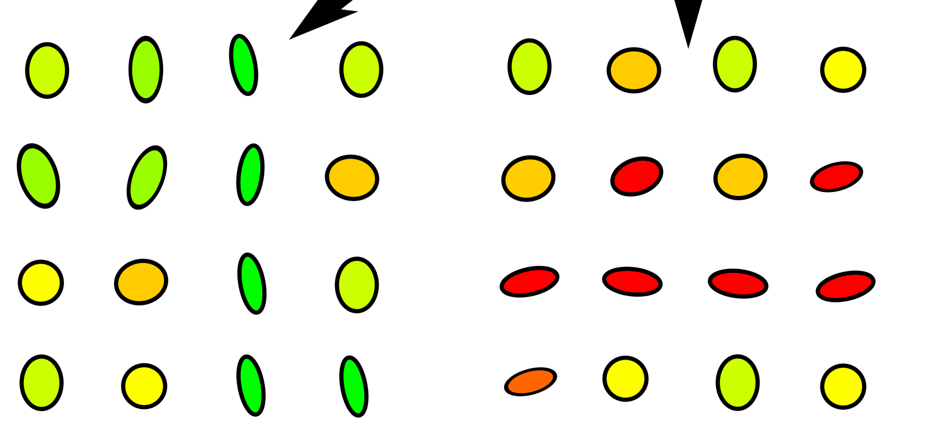
## Motivation

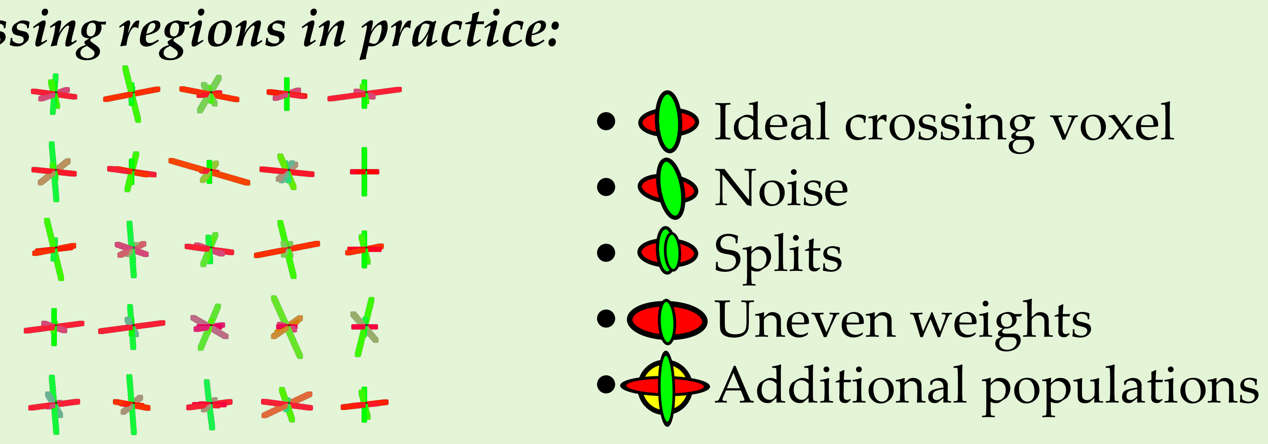



- In this example:
  - Two populations of orientations (population of diffusing water molecules along a fiber orientation)



- ***Goal:*** Infer the populations and smooth them individually
- ***Problem:*** Populations are only locally separable

- ***Crossing regions in practice:***




## Prior Work

- Smoothing in the measurement domain
  - Anisotriopic noise filtering, Sijbers et al. [4]
  - Nonlocal means, Kuurstra et al. [5]
  - Image restoration McGraw et al. [6]
- Single-Tensor de-noising
  - Tabelow et al. [7], Chen et al. [8]
- ***No method extends to multi-tensor models***

### References


[1] Peled, S., Friman, O., Jolesz, F., and Westin, C.-F., "Geometrically constrained two-tensor model for crossing tracts in dwi," Magnetic resonance imaging 24(9), 1263–1270 (2006).

[2] Landman, B. A., Bogovic, J. A., Wan, H., ElShahaby, F. E. Z., Bazin, P.-L., and Prince, J. L., "Resolution of crossing fibers with constrained compressed sensing using diffusion tensor mri," NeuroImage 59(3), 2175–2186 (2012).

[3] Kreher, B., Schneider, J., Mader, I., Martin, E., Hennig, J., and Il'yasov, K., "Multitensor approach for analysis and tracking of complex fiber configurations," Magnetic resonance in medicine 54(5), 1216–1225 (2005).

[4] Sijbers, J., den Dekker, A. J., Van der Linden, A., Verhoye, M., and Van Dyck, D., "Adaptive anisotropic noise filtering for magnitude mr data," Magnetic resonance imaging 17(10), 1533–1539 (1999).

[5] Kuurstra, A., Dolui, S., and Michailovich, O., "Hardi denoising using nonlocal means on s2," in [SPIE Medical Imaging], 83140I–83140I, International Society for Optics and Photonics (2012).

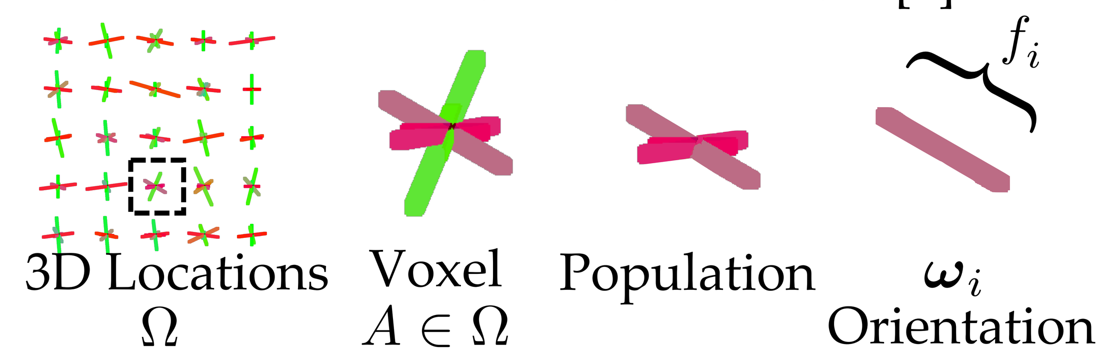
[6] McGraw, T., Vemuri, B.,¨Ozarslan, E., Chen, Y., and Mareci, T., "Variational denoising of diffusion weighted mri," Inverse Problems and Imaging 35(4), 625 (2009).

[7] Tabelow, K., Polzehl, J., Spokoiny, V., and Voss, H. U., "Diffusion tensor imaging: Structural adaptive smoothing," NeuroImage 39(4), 1763–1773 (2008).

[8] Chen, B. and Hsu, E. W., "Noise removal in magnetic resonance diffusion tensor imaging," Magnetic Resonance in Medicine 54(2), 393–401 (2005).


## Approach

- Data after multi-tensor reconstruction: [2]




(Fractional Contribution)
( $\sum_i f_i = 1$ )

- Assumptions:
  - The ***individual populations of orientations are smooth***
  - Locally, each population composes a ***similar fraction*** (e.g. Population X is 70% of voxel A and neighbors)

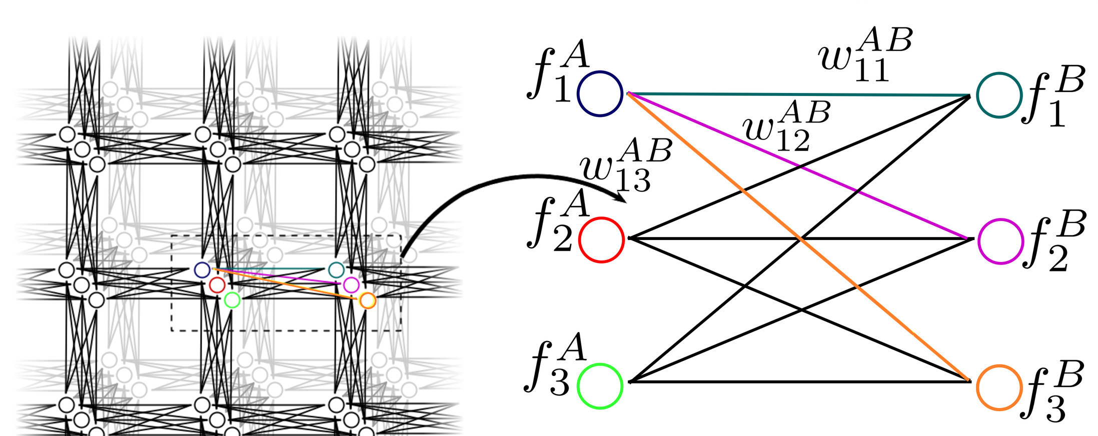
We define a global ***smoothness energy function*** (sum of edges)

$$E = \sum_{A\in\Omega} \sum_{B\in\mathcal{N}_A} \sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij}^{AB} d^2(\omega_i^A, \omega_j^B)$$

- $\mathcal{N}_A$ is the neighborhood of voxel $A$
- $d^2(\omega_i^A, \omega_j^B)$ is the squared distance between the orientations
- $w_{ij}^{AB}$ is a ***weight*** (edge) between orientations (nodes)

- ***Idea: For a given orientation, parts of each neighboring voxel correspond to this and only this orientation***
- ***The weight $w_{ij}^{AB}$ is the "amount shared" between two neighboring orientations***
- ***The energy function becomes a sum of costs between "related" orientations***

- Graph: Between two voxels, all edges leaving a node sum to $f_i$

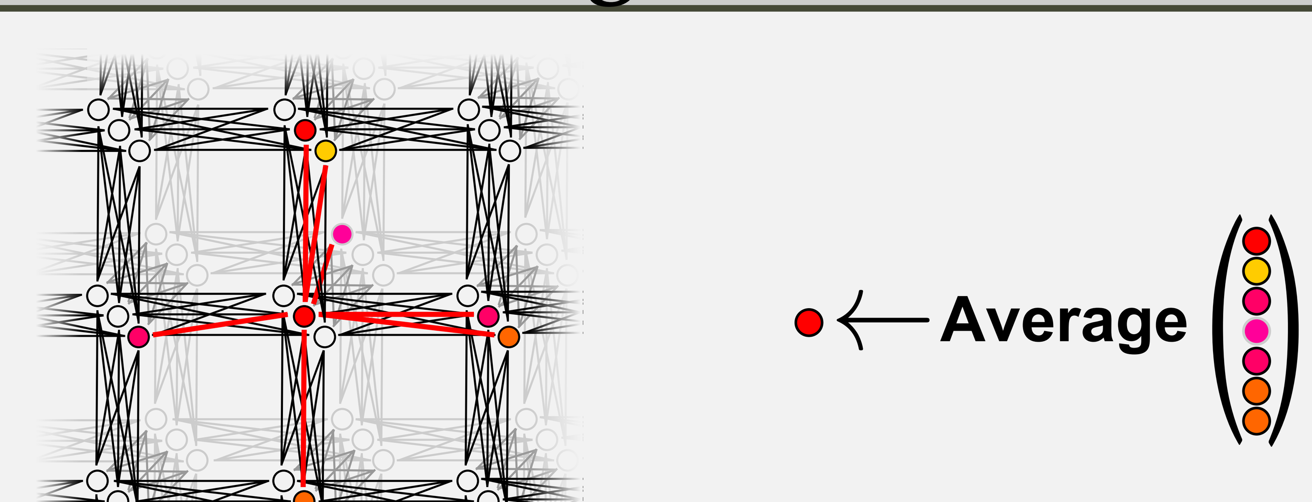



$$\sum_i w_{ij}^{AB} = f_j^B$$

$$\sum_j w_{ij}^{AB} = f_i^A$$

- We can minimize this energy function for $w_{ij}^{AB}$ using linear programming and $\boldsymbol{\omega}_i$ using gradient descent along geodesic
- Solution for $w_{ij}^{AB}$ is the smoothest association into populations

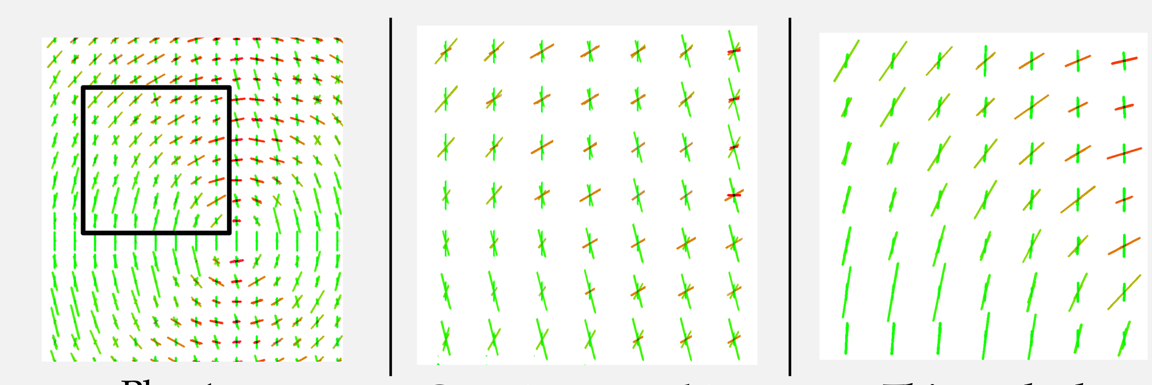
## Algorithm



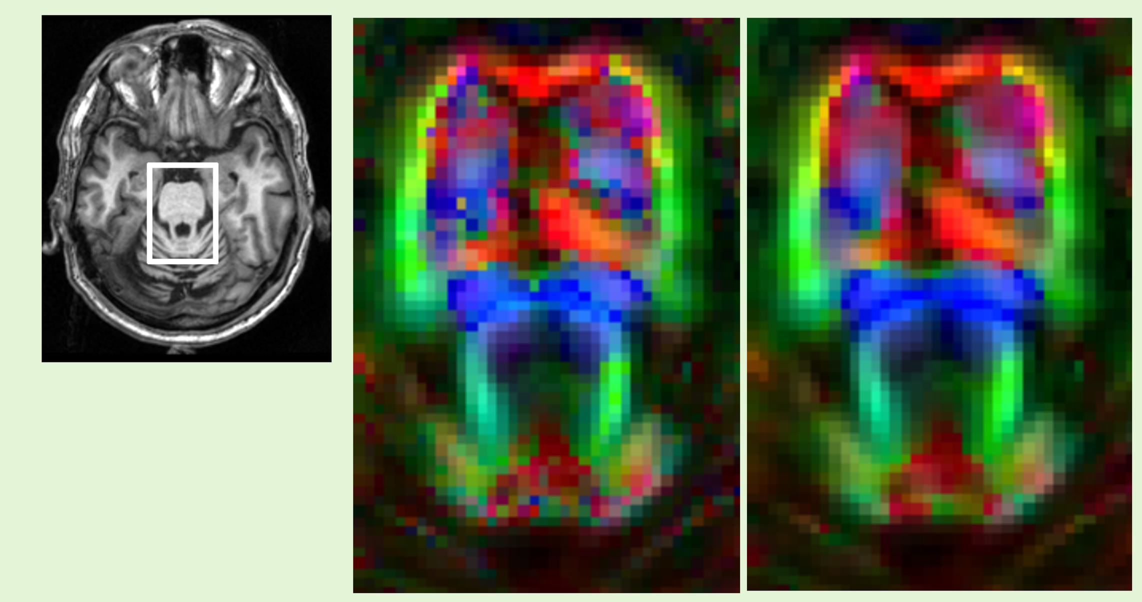

1. Find Correspondences
(Separates down to pairs of voxels)

2. Make an orientation more similar to ***its relatives.***
(Along the correct geodesic)

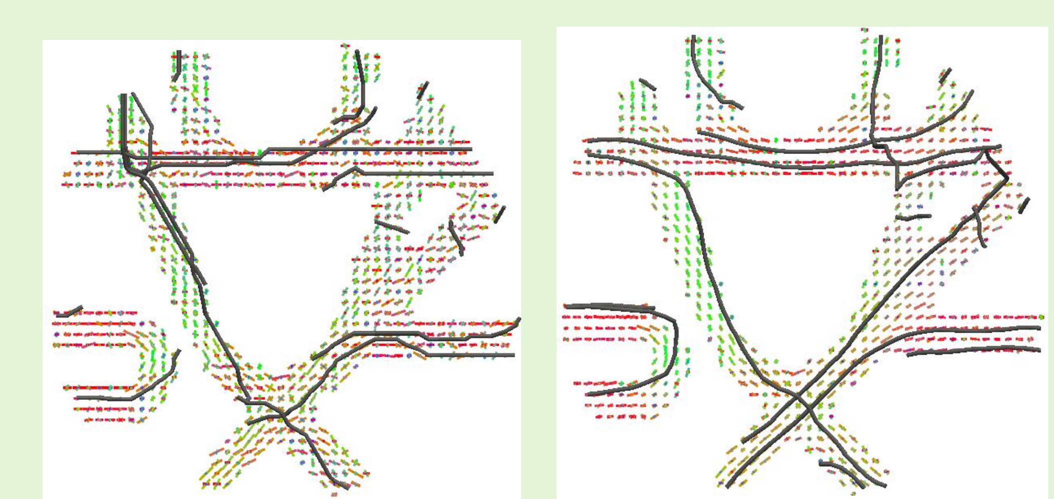
## Results



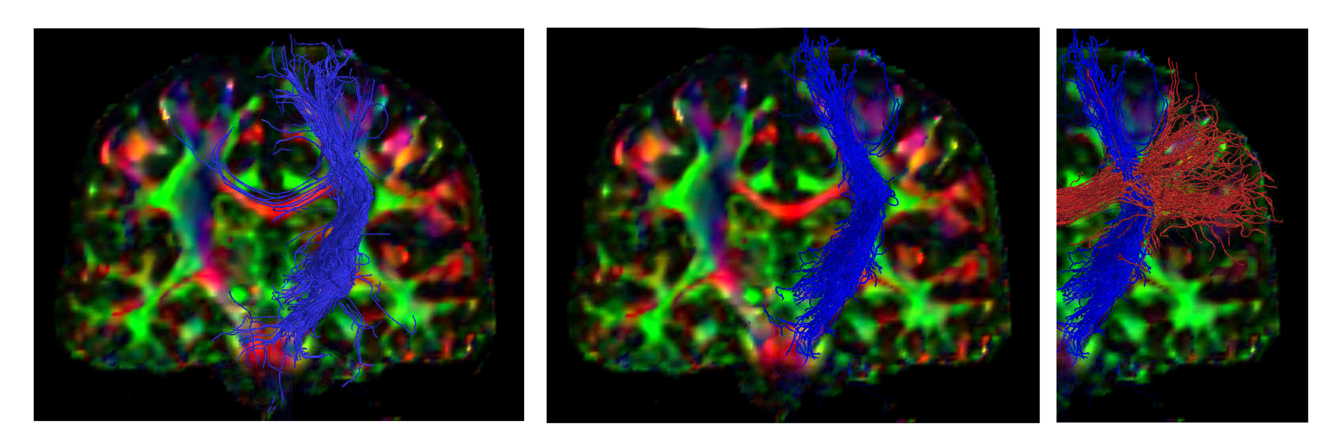
Phantom — Gaussian smoothing (measurement domain) — *This method*



Corticospinal tract and transverse pontine fibers (Axial view) — DTI Color map from Multi-tensor data — *Color map after smoothing*



Simple fiber tracking before smoothing — *Simple fiber tracking after smoothing*



IC Fiber tracking before smoothing — *IC Fiber tracking after smoothing* — *Internal Capsule and Corpus Callosum (Coronal view)*

## Conclusions

- Energy minimization for smoothing multi-tensor data
- If populations are sufficiently smooth, the method smooths individual populations without blending
- Infers populations, potential new appliactions
- May be combined with measurement domain methods


### Acknowledgements

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